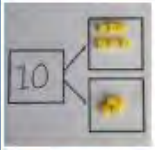
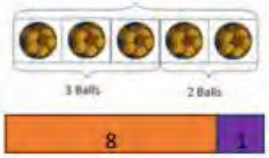
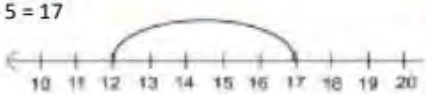
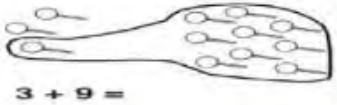
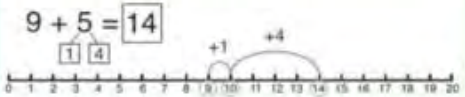
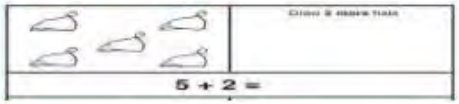
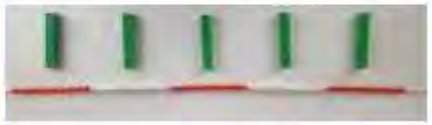
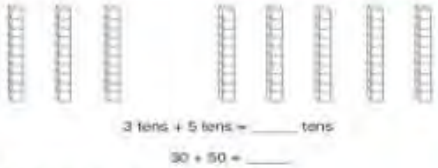
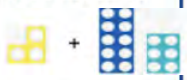
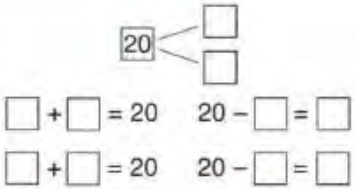
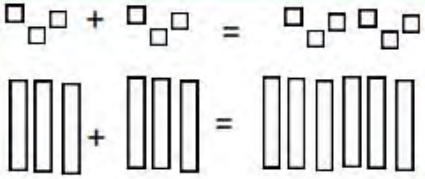
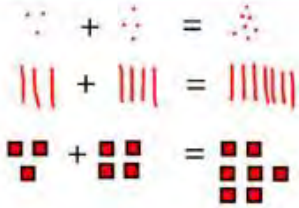
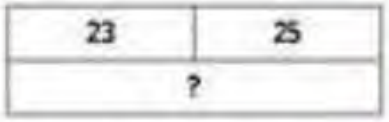
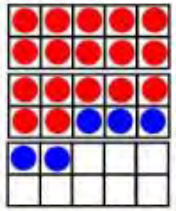
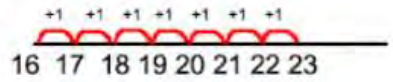
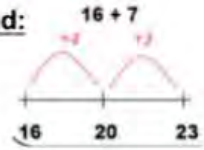
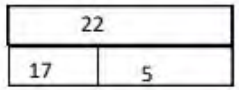
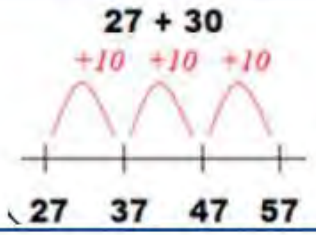
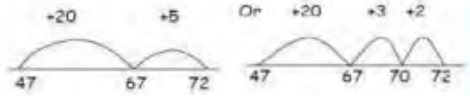
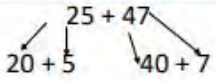
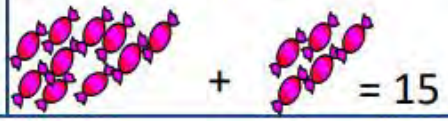
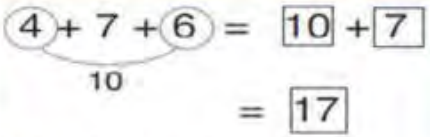
Y1 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 10= 6 + 4 Calculations (number sentences) to be portrayed as **both** __ + __ = __ & __ = __ + __ throughout CPA stages.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

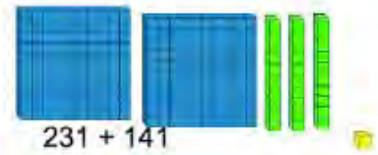
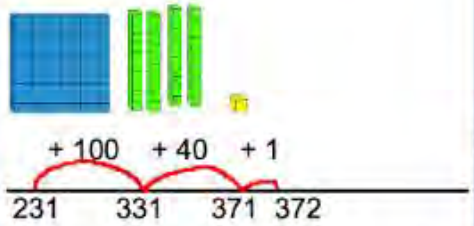
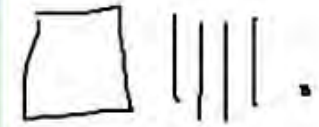
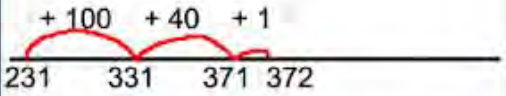
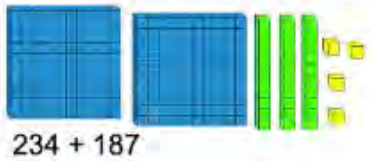
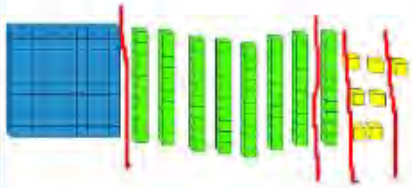
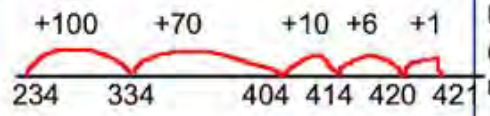
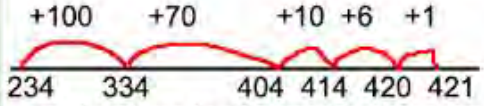
Y2 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 3 tens + 5 tens = ____ tens 30 + 50 = ____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20 	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70  *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

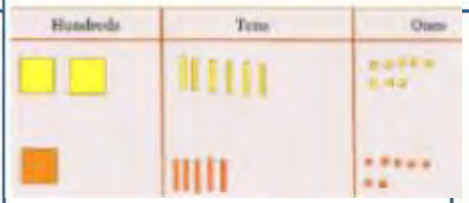
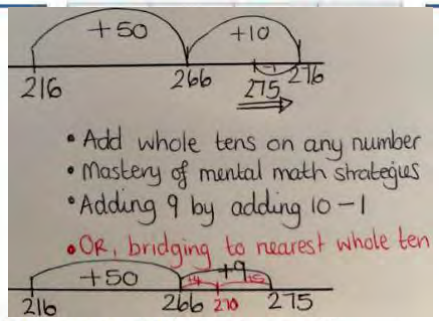
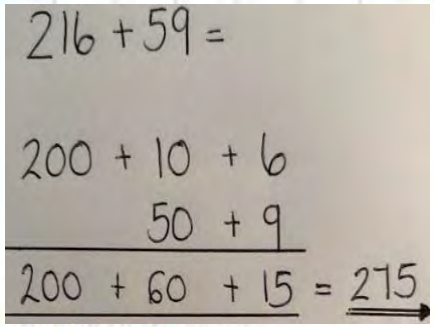
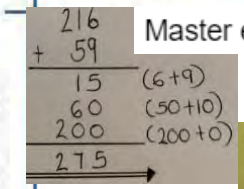
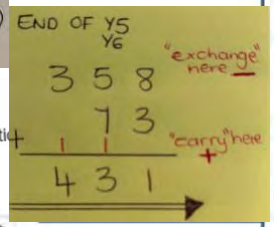
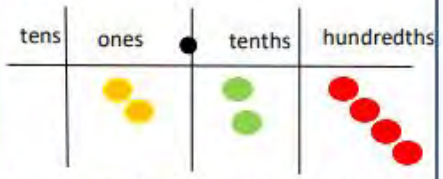
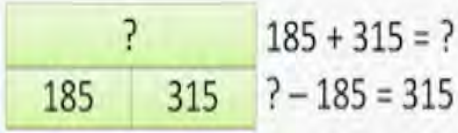
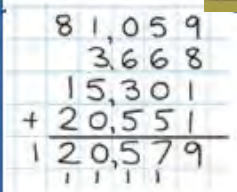
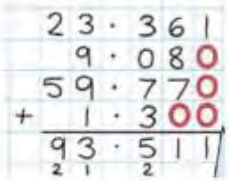
Y2 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones Rem: Chn sh us d Strings.	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	**1st:**  16 17 18 19 20 21 22 23 **2nd:**  16 20 23	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22 – 17 = 5 22 – 5 = 17 **Adding 9** By adding 10 & Subtracting 1 **Adding 11** By adding 10 & Adding 1 more  22 17 5
A ber and tens	 25 + 10 = 35 Explore that the ones digit does not change	 27 + 30 +10 +10 +10 27 37 47 57	27 + 10 = 37 27 + 20 = 47 27 + □ = 57 
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 +20 +5 47 67 72 Or +20 +3 +2 47 67 70 72 Use number line and bridge ten using part whole if necessary.	 25 + 47 20 + 5 40 + 7 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation.  = 15	 4 + 7 + 6 = 10 + 7 10 = 17 Combine the two numbers that make/ bridge ten then add on the third.

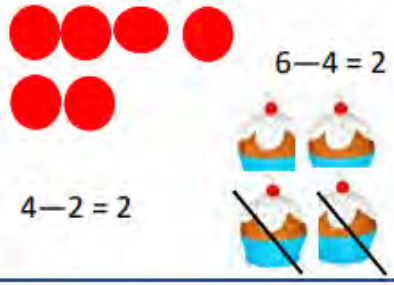
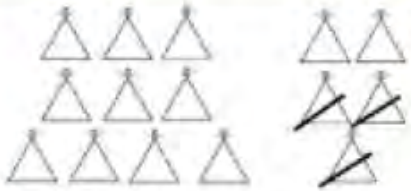
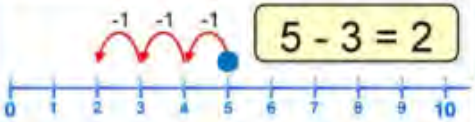
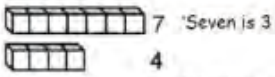
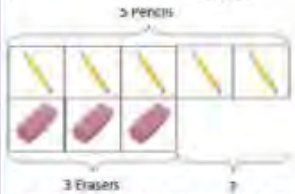
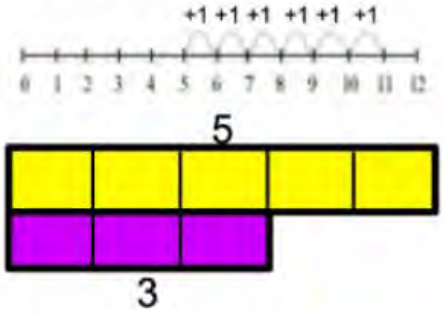
Y3 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Addition - no exchanging Adding two or three 2 or 3 digit numbers Number line taught across CPA - default method for Year 3	  Move in line with a number line when understanding is developed	231 + 141   Use drawings to of concrete resources (HTO) on a numberline. Gradually move to just drawing the jumps.	231 + 141 200 + 100 30 + 40 1 + 1 • Use partitioning & recombinding • Use number line Work until secure • Use bar model representations Pupils identify adding together the Hundreds, the Tens and then the Ones and find the total.
Addition - no exchanging Adding two or three 2 or 3 digit numbers Number line taught across CPA - default method for Year 3 Year 3's should not be doing column method Until secured number lines.	  	234 + 187   Use drawings to of concrete resources (HTO) on a numberline. Gradually move to just drawing the jumps.	234 + 187 200 + 100 30 + 70 + 10 4 + 6 + 1 Pupils identify adding together the Hundreds, then enough Tens to make the next Hundred, then the remaining Tens and then enough Ones to make the next Ten then the final Ones and find the total.

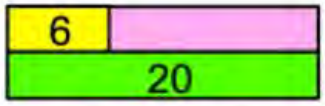
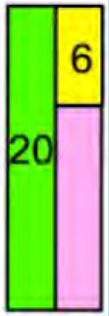
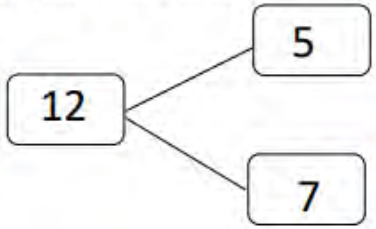
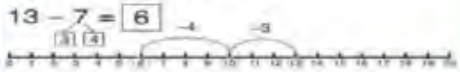
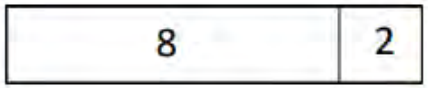
Y4-6 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits 1st. Chn use deans and number lines until secure with addition of 3 digit numbers. 2nd. Chn should partition Numbers: +1's, then 10's and 100's.	 Children use deans, place value counters To add or subtract noticing what happens when carrying or exchange	 Draw representations using pv grid. They can also do this but with drawing the Dienes in columns 	 Relate to money and measures.  Master expanded notation. Note: where we 'carry' When we add. Where We 'exchange' for subtracti 
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.	 Reasoning about addition using Bar model representations Clarity on inverse operations	
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5  	 Insert zeros for place holders. 

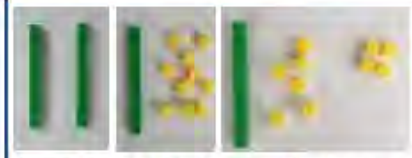
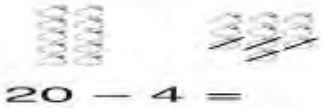
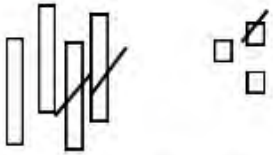
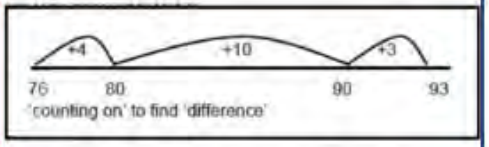
Y1 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters , cubes etc to show how objects can be taken away. 	 15 – 3 = 12 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference. 	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

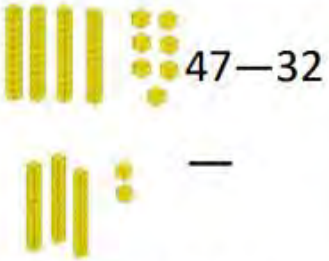
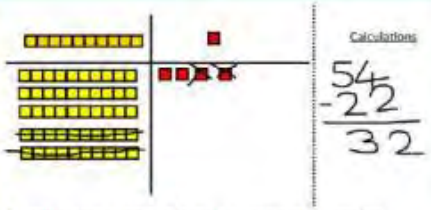
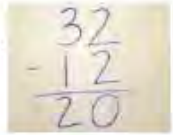
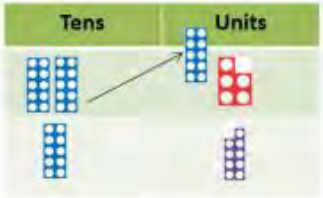
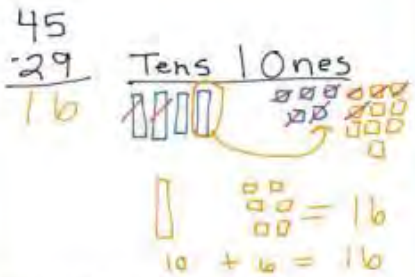
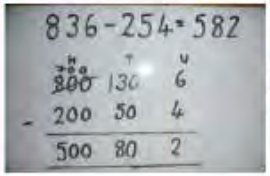
Y1 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use concrete bar model to model the inverse. 	Show the bar model sideways or upside down too!   If 20 is the total and 6 is one of the parts, what is the missing part?	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model Another example to build understanding of bar model	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

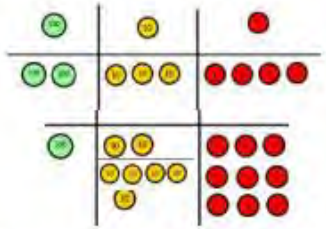
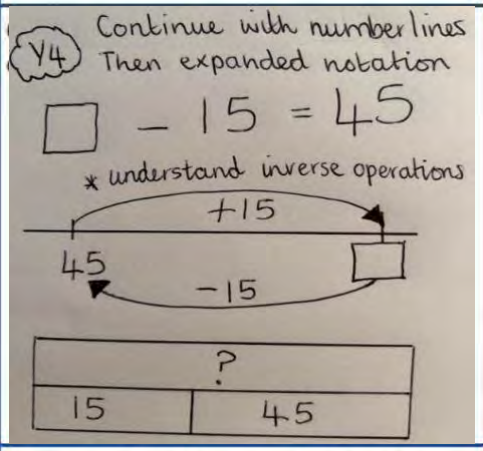
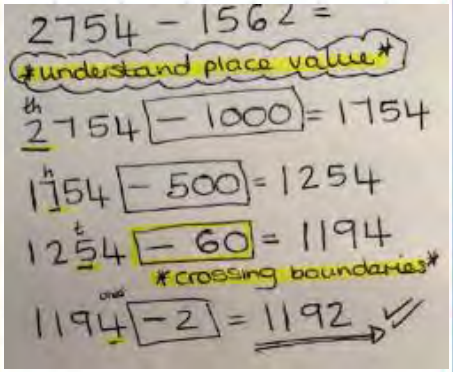
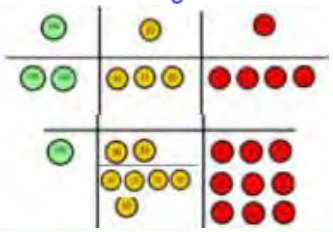
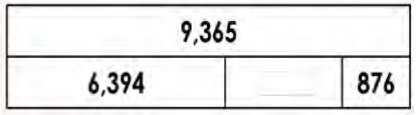
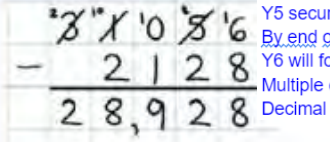
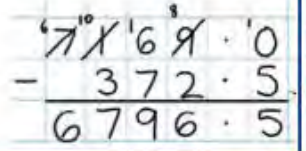
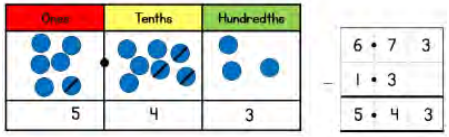
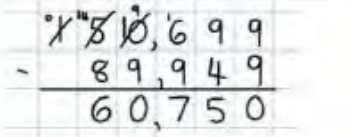
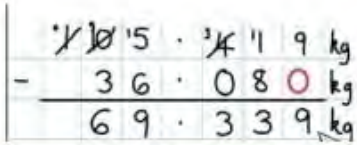
Y2 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 20 — 4 =	20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21    Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22 Y2 chn must acknowledge the position and value of the Numbers in a subtraction Calculation. ASK: Are the numbers far apart or close to one another on a number line? Y2 make choice of efficient method 23 - 19 = finding the difference 23 - 4 = counting backwards
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

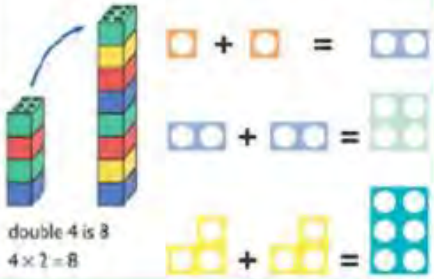
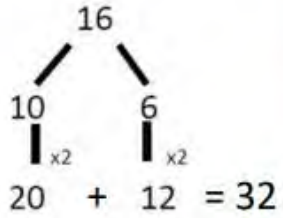
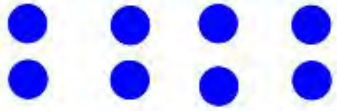
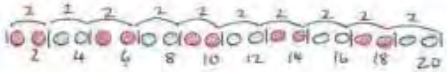
Y3 - SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Numicon to model	 Darw representations to support understanding	47-24=23 40+7 -20+4 20+3 Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off. Teacher draws attention to: • Complementary addition • Crossing boundaries • Crossing over into hundreds	 Begin by partitioning into pv columns Y3 must master subtraction on number line No formal column method Should be taught in Y3 Where there are exchanges. Must keep it in expanded form.

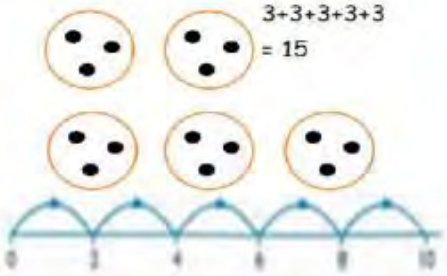
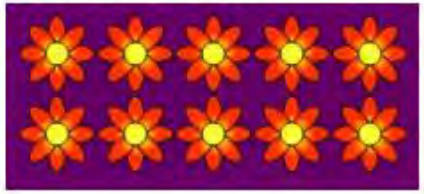
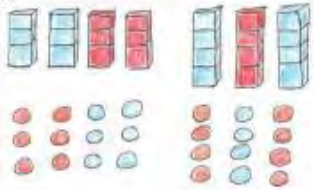
Y4-6 SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.		
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4 Use place value counters To secure understanding of Of exchanges in subtractions. 	Fill in the missing number to complete the bar model. Y5 & Y6  Y5 & Y6 6.73 – 1.3 = 5.43	 Y5 secure exchanges By end of year. Y6 will focus on Multiple exchanges and Decimal subtractions. Use zeros for placeholders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

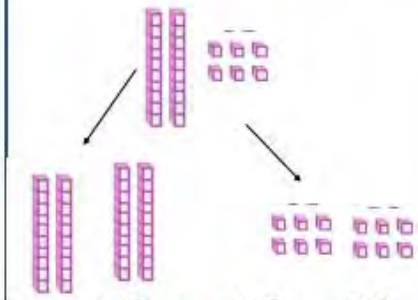
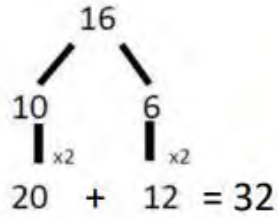
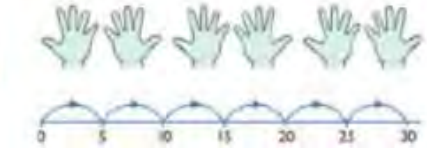
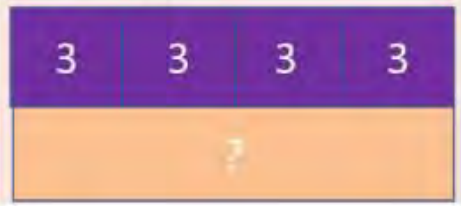
Y1 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers  Double 4 is 8 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10  5, 10, 15, 20, 25 , 30 Number stick
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

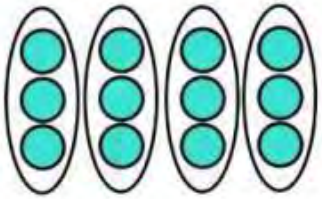
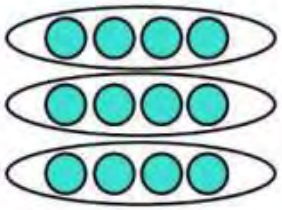
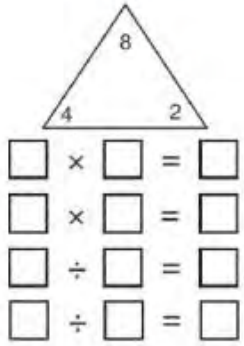
Y1 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	   Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

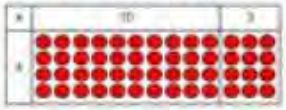
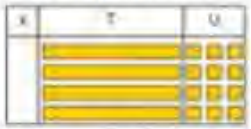
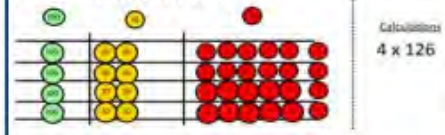
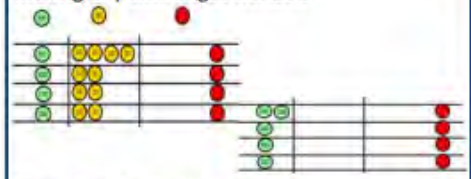
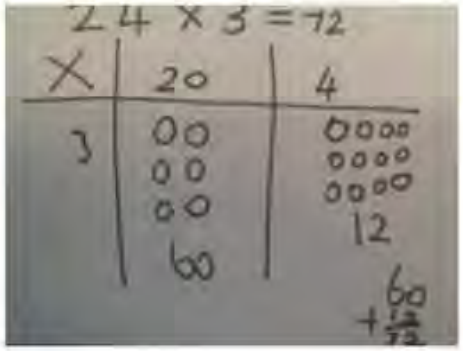
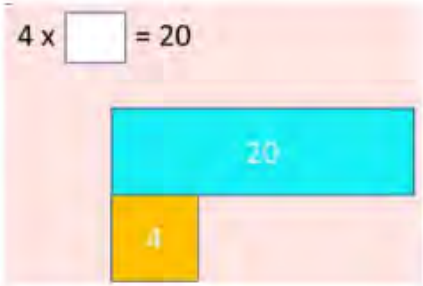
Y2 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling Link doubling to x2	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers  	Partition a number and then double each part before recombining it back together.  16 10 x2 6 x2 20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition) Y2 Must Secure 2x 5x and 10x timestables	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.    3 3 3 3	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 4 × 3 = □

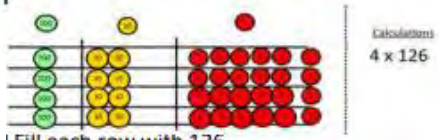
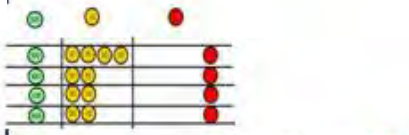
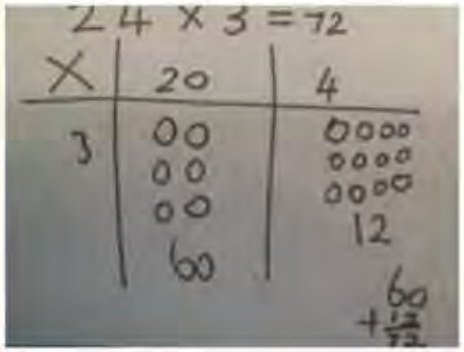
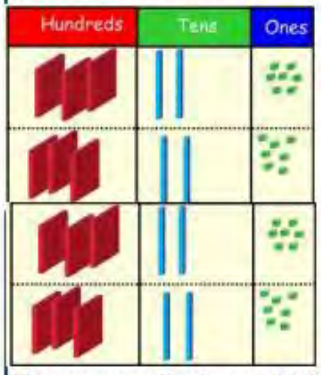
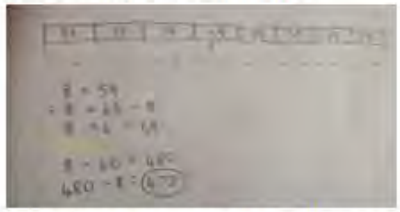
Y2 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

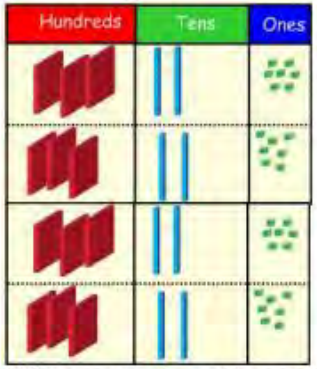
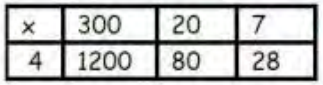
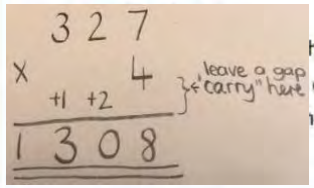
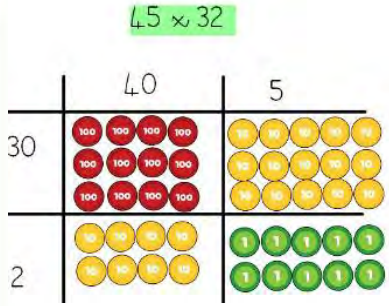
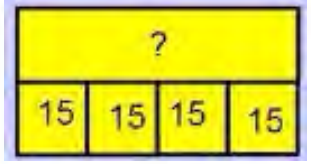
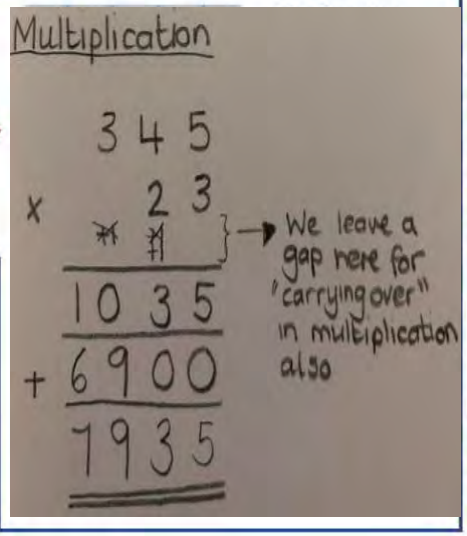
Y3 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method Y3 must secure 2x 5x 10x 11x 3x 6x 4x Timestables	Show the links with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed  Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. | | 10 | 8 | | --- | --- | --- | | 10 | 100 | 80 | | 3 | 30 | 24 |

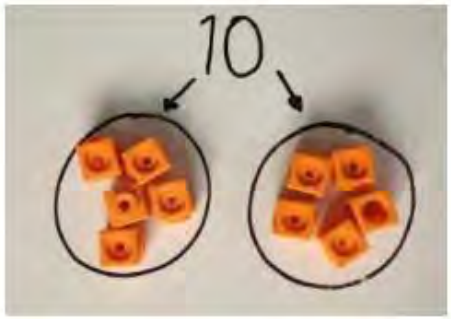
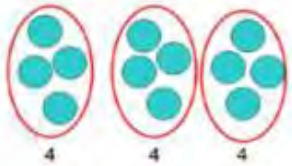
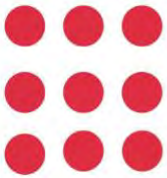
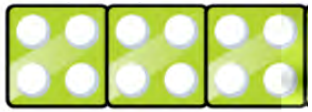
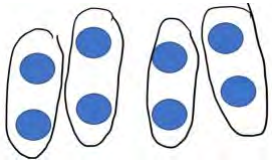
Y4 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Y4 move to 2 digit x 2 digit number. Y4 must secure 7x 8x 9x 12x ALL SHOULD BE SECURED BY END OF Y4 and will be Continuously rehearsed In Y5 and Y6.	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculation: 4 x 126 Fill each row with 126  Add up each colu…ies making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | x | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245 Y4 must secure the grid method BEFORE shown any column Formal methods for multiplication. Only in Summer, IF secure.
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	| x | 300 | 20 | 7 | | 4 | 1200 | 80 | 28 | The grid method my be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308

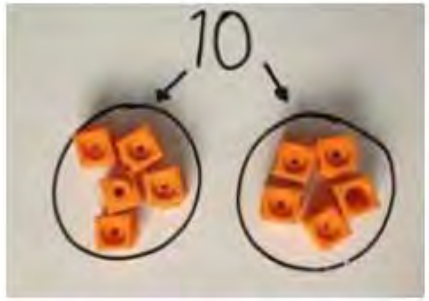
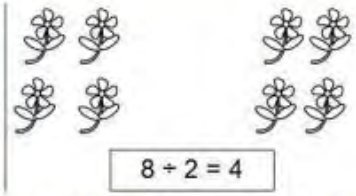
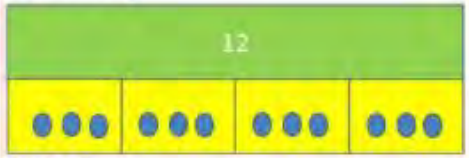
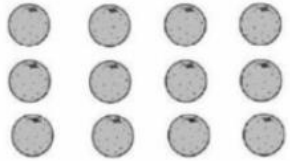
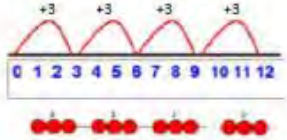
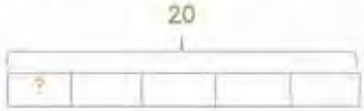
Y5-6 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit. Y5 move to 2, 3, 4 digit x 2 digit Number, ONLY if, Secured expanded column Method of multiplication.	 It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642		327 x 4 28 80 1200 1308  This will lead to compact method.
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside. 	  Continue to use bar modelling to support problem solving	

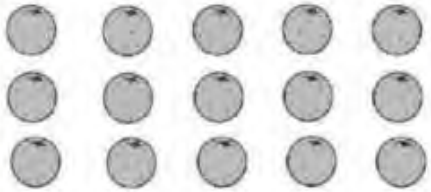
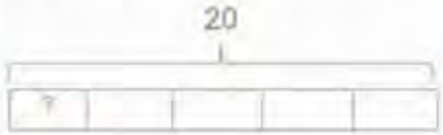
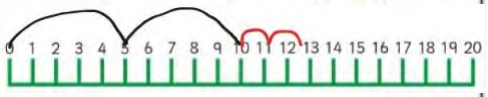
Y1 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing - Grouping quantities - Sharing small quantities - Count in 2's and 5's - Adding equal groups - Understand repeated addition is multiplication. - Making arrays *'Dividing' to be used interchangeably with the term 'sharing'	  Grouping equal quantities  Sharing small quantities I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4  3 + 3 + 3 = 3 x 3 = Adding equal groups	12 shared between 3 is 4  Understand repeated addition is multiplication. Share 8 object into 4 equal groups  Start of mark making in division questions

Y2 DIVISION ÷

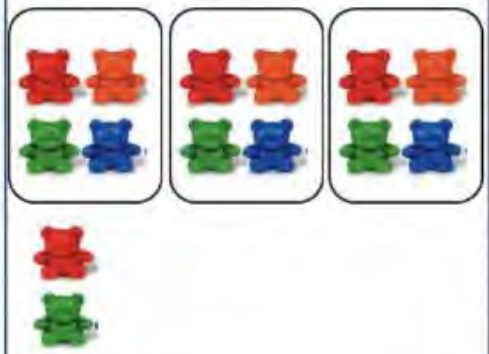
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4  3 + 3 + 3 + 3 = 12 3 x 4 = 12 12 divided by 4 is 3 Begin to make simple statements about an array. Begin to make simple Connections between Multiplication and division.
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Y3 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4 12 ÷ 5 = 2 r 2  Calculating division on A number line showing Remainders using smaller Number - link to timetable And discuss odd and even Number in relation to division

Y3 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.	Complete written divisions and show the remainder using r.

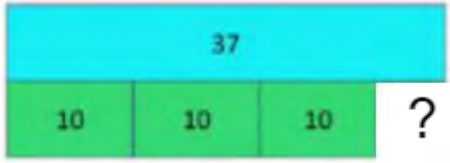


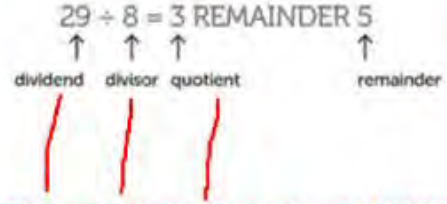


Draw dots and group them to divide an amount and clearly show a remainder.

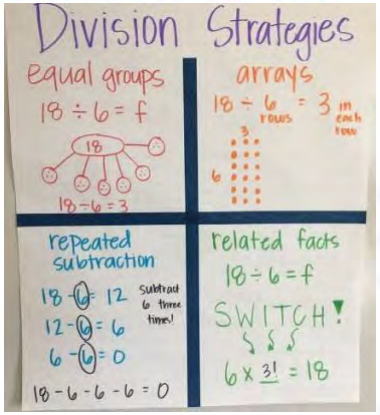


Use bar models to show division with remainders.

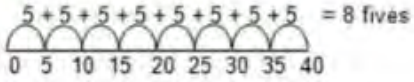




This vocab isn't used until UKS2 but can be a useful reference if appropriate for teacher and/or pupil

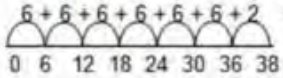


Example without remainder
40 ÷ 5
Ask "How many 5s in 40?"



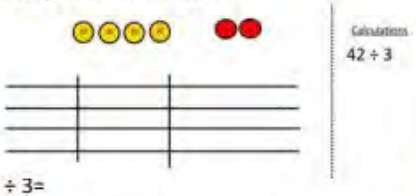
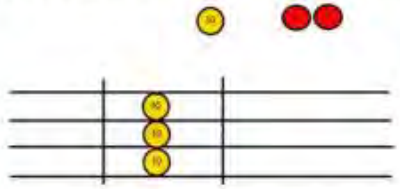
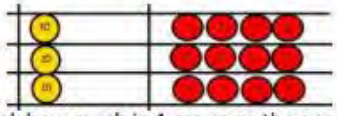
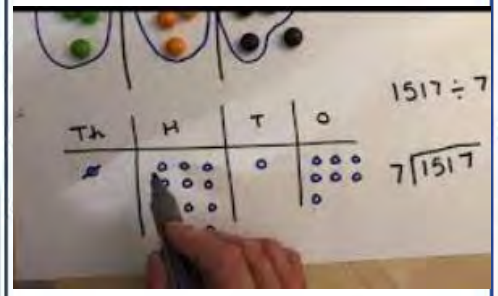
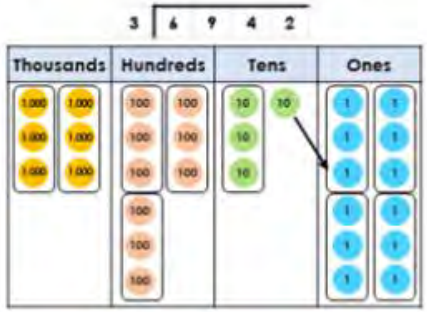
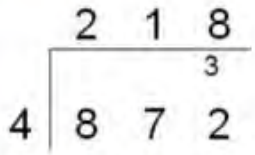
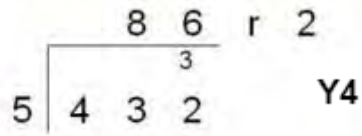
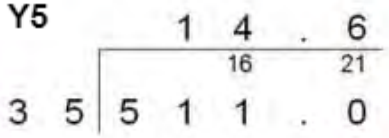
Y3 eventually are calculating division on a number line showing Remainders using larger Number as a method.

Example with remainder
38 ÷ 6



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Y4-6 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division Y4 Master short Division & Have thorough Understanding of short division method in the bus Stop shelter. * See concrete eg. Y5 Master short Division method With a 2 digit divisor. Y5 Reinforce understanding Using concrete eg's. Fact boxes to be Introduced as a supportive strategy.	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. **1. Use the place value chart to complete the calculation.**  72 ÷ 3 = 24 60 ÷ 3 = 20 12 ÷ 3 = 4	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Y4 Finally move into decimal places to divide the total accurately. Y5  Fact Box 35 x1 = 35 35 x 2 = 70 35 x 4 = 140 35 x 10 = 350 35 x 5 = 175 35 x 6 = 210 0663 r 5 8)5³3⁵0²9

Long Division

Y6

DIVISION ÷

Step 1—a remainder in the ones

```
    h t o
    0 4 1 R1
  ---------
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
      th h t o
      0 4 0 0 R7
    -------------
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Y6 DIVISION ÷

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Y6 DIVISION ÷

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8	h t o 1 2)2 7 8 -2 0	h t o 1 8 2)2 7 8 -2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7	h t o 1 3 2)2 7 8 -2 0 7 - 6 1	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 - 1 8 0	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.